An extended Bernese Moon gravity field and first tidal Love number k2 solution from GRAIL

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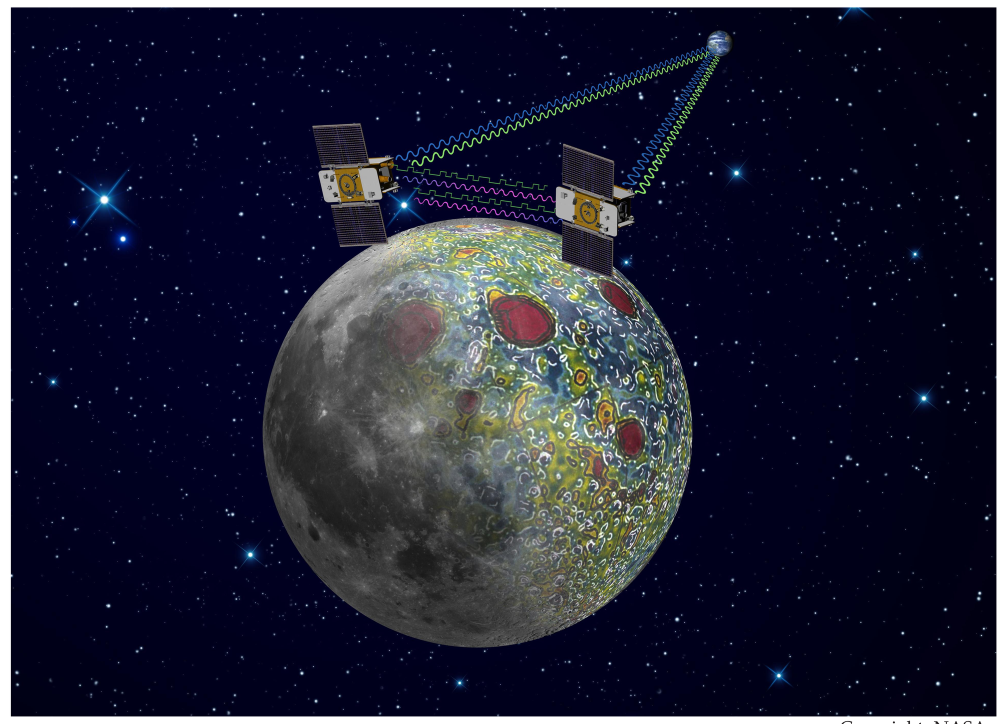
Introduction

To determine the gravity field of the Moon, the two satellites of the NASA mission GRAIL (Gravity Recovery and Interior Laboratory) were launched on September 10, 2011 and reached their lunar orbits in the beginning of 2012 (**?**). The concept of the mission was inherited from the Earth-orbiting mission GRACE (Gravity Recovery and Climate Experiment) as the key observations consisted of ultra-precise inter-satellite Ka-band range measurements. Together with the one- and two-way Doppler observations from the NASA Deep Space Network (DSN), the GRAIL data allows for a determination of the lunar gravity field with an unprecedented accuracy for both the near- and the far-side of the Moon. The latest official GRAIL gravity field models contain spherical harmonic (SH) coefficients up to degree and order 1500 (**??**).



Based on our experience in GRACE data processing, we have adapted our approach for gravity field recovery, the Celestial Mechanics Approach (CMA, Beutler et al., 2010), to the GRAIL mission within the Bernese GNSS Software. We use the level 1b Ka-band range-rate (KBRR) data as well as two-way Doppler observations from the DSN. Earlier results using KBRR data along with JPL-provided GNI1B position data (Arnold et al., 2015) are also presented. The following results are based on the release 4 data of the primary mission phase (PM, 1 March to 29 May 2012).

The Celestial Mechanics Approach (CMA)

The idea of the CMA is to rigorously treat the gravity field recovery as an extended orbit determination problem. It is a dynamic approach allowing for appropriately constrained stochastic pulses (instantaneous changes in velocity) to compensate for inevitable model deficiencies. For each satellite, the equations of motion to be solved read as $\ddot{\mathbf{r}} = \mathbf{a}_G + \mathbf{a}_P$, where $\mathbf{a}_G = \nabla V$ denotes the acceleration due to the gravity potential V, which we parametrize in terms of the standard SH expansion, and $\mathbf{a}_P$ denotes the sum of all perturbing accelerations. We consider 3^{rd} body perturbations according to JPL ephemerides DE421, forces due to the tidal deformation of the Moon and relativistic corrections. We do not yet model direct or indirect solar radiation pressure explicitly.
All observations contribute to one and the same set of parameters, which are simultaneously estimated. Depending on the setup, these are chosen amongst:

- Orbits: Initial conditions every 24h; constant and once-per-revolution (opr) accelerations in R,S,W (radial, along-track, out-of-plane); stochastic pulses in R,S,W estimated periodically (when observations are available). Their spacing has to be chosen as a compromise between making up for model deficiencies and not absorbing too much of the gravity signal.
- Static gravity field: The coefficients of the SH expansion up to the chosen degree and order.

Doppler data processing in the Bernese Software

Besides the inter-satellite KBRR link, GRAIL orbit and gravity field determination is based on its Doppler tracking by several Earth-based stations of the DSN for the absolute positioning of the probes. Both one-way X-band and two-way S-band are available with an accuracy of 0.03 mm/s (~ 2 mHz) and 0.2 mm/s (~ 6 mHz), respectively.
We process Doppler observations using new implementations in the Bernese GNSS software (Bertone et al., 2015). Our modeling is based on the reference (Moyer, 2000) guidebook and follows the schema in Fig. 2.

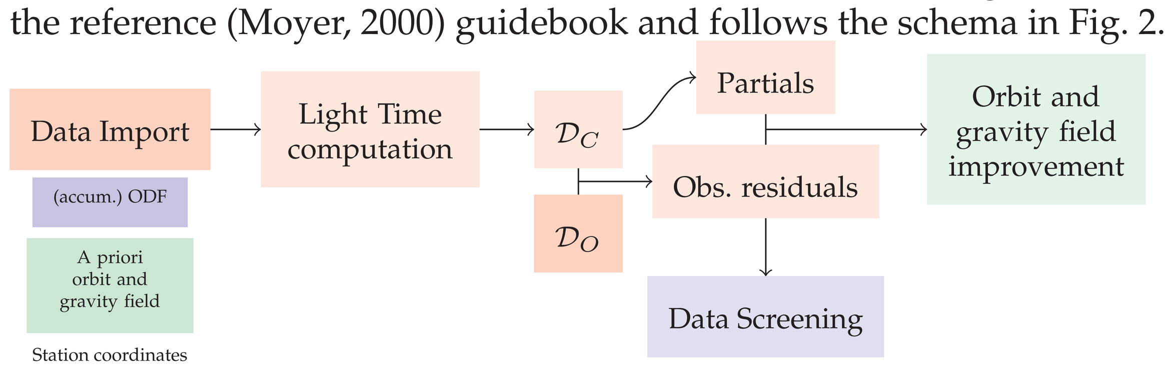


Figure 1: Processing flow of Doppler data, recently implemented in the Bernese (GNSS) Software. Doppler observations $\mathcal{D}_O$ from Orbit Determination Files (ODF) are imported to our internal format and eventually accumulated to the desired integration time. Orbit integration from a priori initial elements and parameters and an accurate modeling of light propagation are used to compute simulated Doppler $\mathcal{D}_C$ and hence Doppler residuals. The latter can be used to screen the observations or, along with the corresponding variational equations, to improve the "a priori" elements in an iterative orbit and gravity field improvement process.

We use the positions provided by the GRAIL navigation team as initial conditions for each daily arc and perform an orbit integration with the force model presented in the previous section. The initial orbital elements and, possibly, dynamical and stochastic parameters are then adjusted to the Doppler data (accumulated over 10 s) using a classical least-square procedure. Observations are screened for outliers by setting a threshold on the residuals, excluding selected time windows and by applying an elevation cutoff at 25°.

Doppler and KBRR orbit determination

Several tests were performed to show the impact of different background fields and parametrizations (dynamic or pseudo-stochastic) on the improved orbits. Fig. 2 (left) shows one- and two-way Doppler residuals for GRAIL-A as well as the daily RMS of Doppler residuals over the PM phase (right). One-way Doppler residuals is slightly noisier because of the estimate of additional clock parameters and of a problem on the on-board clock in the initial phases of the mission.

Doppler and KBRR data are combined on the Normal EQuation (NEQ) level using a weighting appropriate to their relative accuracy ($1:10^8$). The resulting daily NEQs are then inverted to solve for the improved orbital parameters.

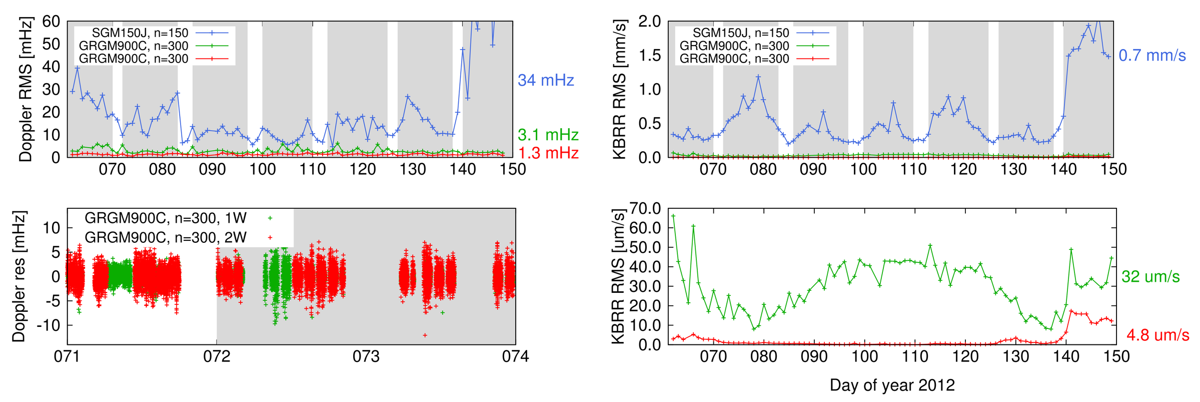


Figure 2: Left: (Top) Daily RMS of GRAIL-A one- and two-way Doppler residuals using GRGM900C (up to d/o 300) and SGM150J as background gravity fields. (Bottom) One- and two-way Doppler residuals over doy 2012 071-073 based on GRGM900C (up to d/o 300). Right: Daily RMS of KBRR residuals over the PM phase. Bottom plot is a zoom of the upper one.

Compared to the expected noise level of around 0.05 μm/s, the KBRR residuals are still relatively large (1 μm/s when excluding the last week of the PM phase). Radiation pressure modeling is crucial since the chosen parametrization is not able to fully compensate the deficiency. However, we show that a good residuals level can be reached without a complex modeling of non-gravitational forces when using pseudo-stochastic pulses.

Gravity field and k_2 from Doppler and KBRR data

The orbits determined in the first combined orbit determination serve as a priori information for a common orbit and gravity field estimation based on daily arcs. All solutions are computed using GRGM900C (up to d/o 660) as background field, 30' pulses in S and W directions, a constant acceleration in S and opr accelerations in R. A classical least-squares adjustment is used. The daily normal equation systems (NEQs) are stacked to weekly, monthly and finally three-monthly NEQs (the whole PM phase), which are then inverted.
Also, some preliminary experiment to estimate the tidal Love number k_2 has been performed, as shown in Table 1.

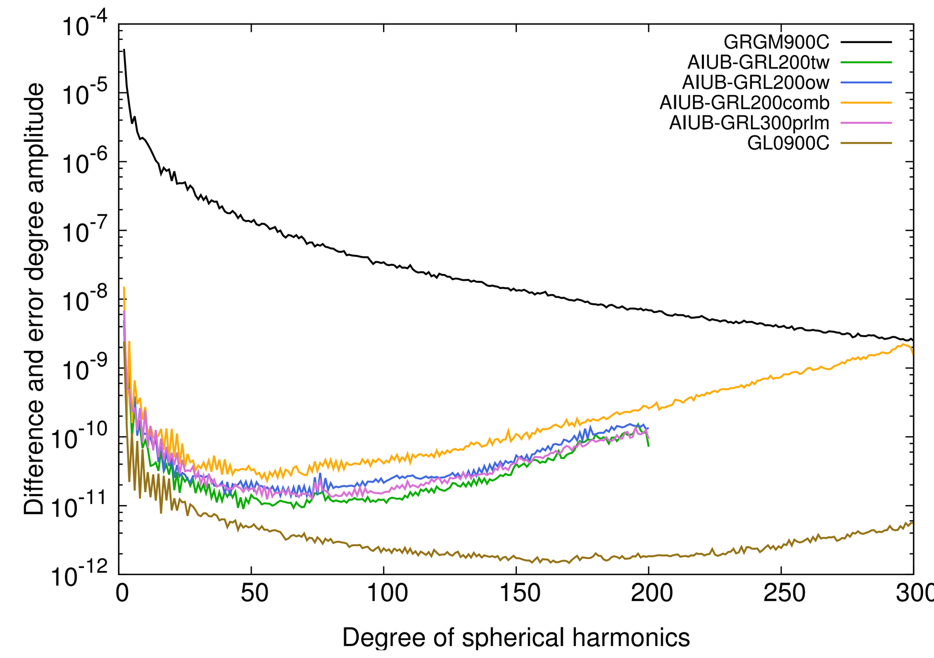


Figure 3: Difference degree amplitudes (solid) and formal errors (dashed) of degree-200 and -300 solutions based on the a priori field GRGM900C (up to d/o 660) w.r.t. GRGM900C. The blue solution is based on one-way Doppler and KBRR data while the green one on two-way Doppler. The purple solution is a combination of one-way and two-way Doppler based daily NEQs with equal weighting. The gold line is a preliminary two-way Doppler based d/o 300 solution. Finally, the brown curve shows the difference between the two american solutions GRGM900C and GL900C.

Solution	$k_2 \pm \sigma$
AIUB-300prlm	0.023958 ± 0.000101
AIUB-200ow	0.022661 ± 0.000082
AIUB-200tw	0.024147 ± 0.000069
GRGM900C	0.024116 ± 0.000108

Table 1: Moon k_2 Love number solutions estimated with the gravity field solutions above.

Gravity field from Kaguya a priori field SGM150J

In order to compute a "truly" independent gravity field of the Moon, the SELENE gravity field SGM150J is used as a priori field for a d/o 300 solution. The whole spectrum (excepted the lowest degree) is improved after a single iteration even if additional iterations are needed to fully exploit the KBRR information.

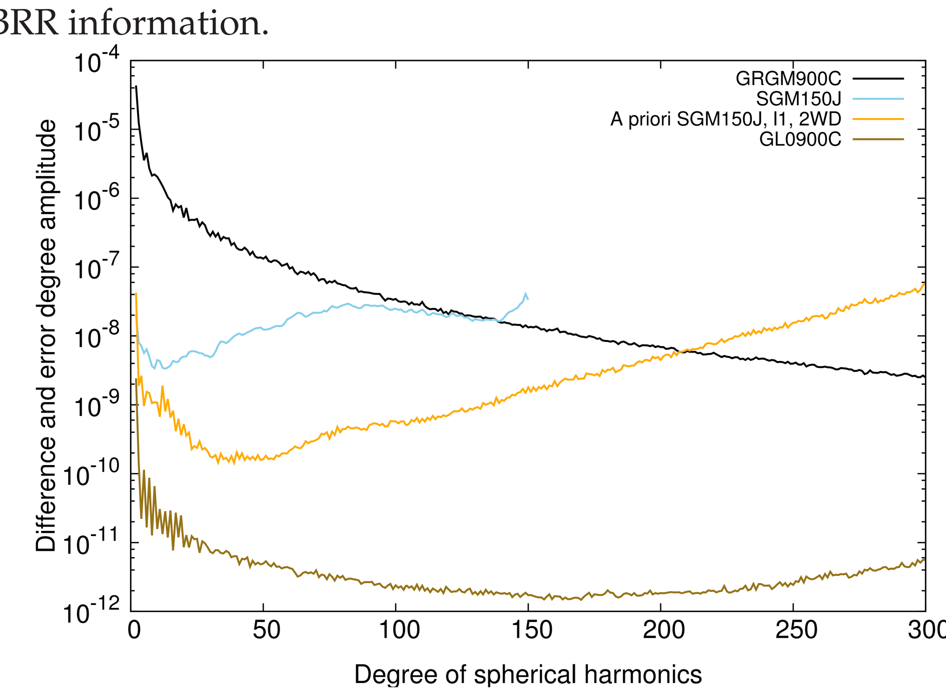


Figure 4: Difference degree amplitudes (solid) and formal errors (dashed) of a d/o 300 solution based on SGM150J (red) and of SGM150J itself (blue) w.r.t. GRGM900C.

Triangle plots show individual coefficient differences between GRGM900C and SGM150J (left) and our d/o 300 solution (right), respectively. All coefficients are improved up to d/o 150 and one can notice that the error on the higher coefficients is mostly due to the sectorial terms (because of the GRAIL setup relying on the along-track KBRR intersatellite link).

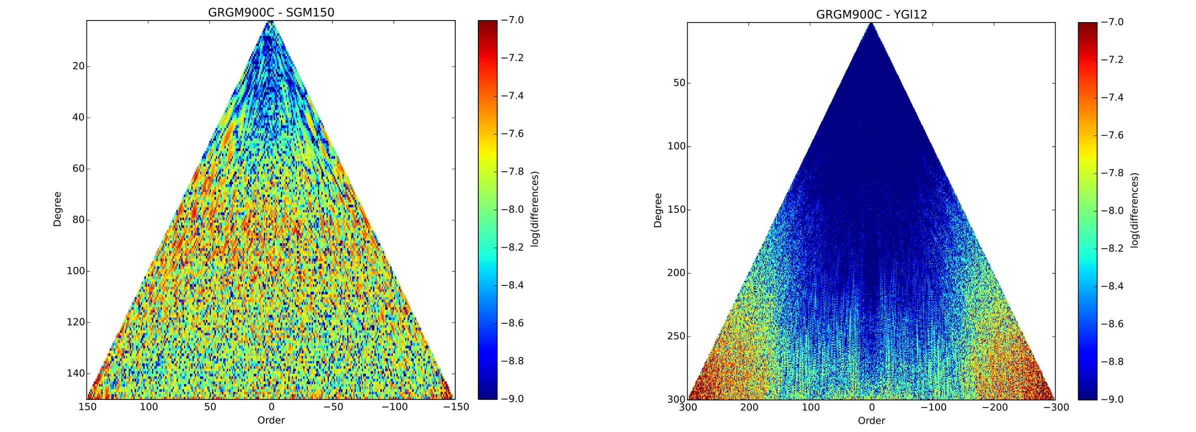


Figure 5: Triangle plots of the SELENE (SGM150J) gravity field (left) and of our d/o 300 solution based on SGM150J (right) w.r.t. GRGM900C.

Figure 6 (left) shows that KBRR residuals are significantly reduced when using our SGM150J derived d/o 300 gravity field instead of the SGM150J field itself. In particular, the large differences between the "face-on" and "edge-on" days almost disappears. Also, we see a strong correlation between the amplitude of out-of-plane pulses and the angle of the orbital plane with the line-of-sight.

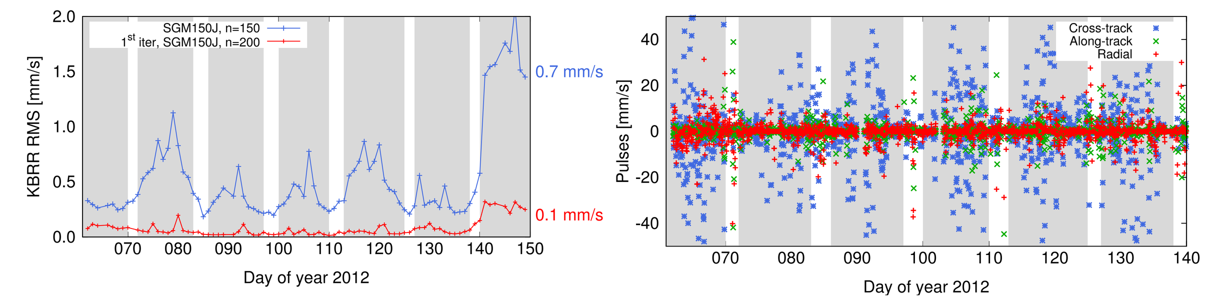


Figure 6: Left: Daily RMS values of the KBRR residuals in the combined (Doppler and KBRR) orbit solution based on the SELENE (SGM150J) gravity field (blue) and on our extended d/o 300 solution (red) based on SGM150J. Shaded days represent geometries when less than 80% of the orbit is visible from Earth.

We recently adapted our processing to allow for larger solutions and are now studying an adapted parametrization to start the iteration process which should finally lead to an improved fully independent solution.

Conclusions

- The adaption of the CMA from GRACE to GRAIL allows for lunar gravity fields obtained entirely within the Bernese GNSS Software.
- We present our first d/o 300 solutions for the lunar gravity field computed from original GRAIL one-way and two-way Doppler and KBRR data, hence showing our ability to extend our activities to the analysis of planetary missions data.
- Our gravity field solutions are so far computed without explicitely modeling non-gravitational forces and demonstrate the potential of pseudo-stochastic orbit parametrization.
- **Outlook**: extended gravity field solutions (up to d/o 420), optimal combination of one-way and two-way Doppler.

References

Arnold et al. (2015) *GRAIL gravity field determination using the Celestial Mechanics Approach*. Icarus 261:182-192

Bertone et al. (2015) *Doppler Orbit Determination of Deep Space Probes by the Bernese GNSS Software*. Proc. of the 25^{th} ISSFD.

Beutler et al. (2010) *The celestial mechanics approach: theoretical foundations*. J Geod 84:605-624 and *The celestial mechanics approach: application to data of the GRACE mission*. J Geod 84:661-681

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